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Direct integration of gradually varied flow equation in parabolic channels

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ABSTRACT

The direct integration method is used to compute water surface profiles of gradually varied flow (GVF) in a prismatic open channel. No closed-form solution is available for the GVF equation in the case of general parabolic channels. Open channels with parabolic cross-sections are often a quite good approximation of the real geometry of natural rivers. Technology is also available for constructing this shape of channels. In the present study, by applying the Manning formula, a semi-analytical solution to compute the length of the gradually-varied-flow profile for parabolic channels is developed. The proposed solution uses a single step for the computation of water surface profiles and, as such, provides an accurate and yet simple way to compute GVF flow profiles; thus, it should be of interest to practitioners in the hydraulic engineering community.

1. Introduction

Almost all major hydraulic engineering activities involve the computation of the gradually varied flow (GVF) profile. The GVF profiles through open channels are also of considerable importance to the water structures engineers. The computation of the GVF profile has been an important topic of interest during the last century. The most widely used methods for computing the water surface profiles in open channels are classified into two general groups, that is, step methods and direct integration methods. The step methods [1] can be utilized when two flow depths are given and the distance between them is required (called as a direct step method) or when the flow depth at a specified location is required (known as a standard step method). The direct integration methods entail the integration of the GVF dynamic equation and may be carried out using analytical, semi-analytical, or numerical procedures. Numerical integration of the GVF governing equation is mainly used in natural or non-prismatic channels. In some prismatic artificial channels, the direct integration is straightforward and total length of the profile can be calculated using a single step. The focus of this research is on analytical or semi-analytical integration of the GVF governing equation.

Direct integration of the GVF dynamic equation has been receiving considerable attention by [2–11]. In spite of a lot of investigations on the GVF, there is no closed-form (analytical or semi-analytical) solution for this equation for the case of parabolic channels. The parabolic channel has many advantages in practice [12–14] (a) parabolic cross-sections are often a quite good approximation of the real geometry of natural rivers; (b) parabolic sides improve slope stability by reducing the problems caused by water seepage; (c) unlined channels tend to approximate a parabolic shape after a long period; (d) technology is available for constructing this shape of channels; and (e) parabolic channels do not have sharp edges at which cracks may occur due to stress concentration.

In the present study, a semi-analytical integration is used to compute the flow profiles in a parabolic channel. The following sections present the geometric properties of a parabolic section, the governing equation and the proposed semi-analytical integration procedure.

2. Geometric properties of a parabolic section

A parabolic channel (Fig. 1) is described as follows:

$$Y = kX^2 \tag{1}$$

where Y = ordinate; X = abscissa; and k = parameter.

The flow area of the channel, A, for the flow depth, y, can be computed as

$$A = 2\int_0^{\sqrt{\frac{y}{k}}} (y - kX^2)\mathrm{d}X = \frac{8}{3} zH^2\eta^{3/2} \tag{2}$$

where z = side slope (1 vertical to z horizontal) at the bank level, H = channel depth and $\eta = y/H$ = dimensionless flow depth ($0 < \eta < 1$). The wetted perimeter, P, is also obtained as follows:

$$P = 2\int_0^{\sqrt{\frac{y}{k}}} \sqrt{1 + 4k^2X^2}\mathrm{d}X = 2z^2H\left\{\sqrt{\frac{\eta}{z^2} + \frac{\eta^2}{z^4}} + \ln\left(\frac{\eta^{1/2}}{z} + \sqrt{1 + \frac{\eta}{z^2}}\right)\right\}. \tag{3}$$

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