Analytical solution of specific energy and specific force equations: Trapezoidal and triangular channels

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ABSTRACT

The specific energy and specific force equations have many applications in open-channel flow problems. At present, these equations have analytical solution only for rectangular channels. Trial and error procedure also graphical solutions are the existing methods of solving these equations. No analytical solutions are available in the technical literature for these equations in trapezoidal and triangular channels because it is presumed that these equations are quintic equations. The inversion of such equations consists of finding the roots of quintic equations. In the current study for a given channel geometry and discharge, the subcritical (supercritical) depth is analytically found in terms of the other supercritical (subcritical) depth. For this purpose, by considering physically meaningful domains, a quintic equation has been reduced to a quartic equation. In the next step, this quartic equation has been converted to a resolvent cubic equation and two quadratic equations. This research shows these steps clearly to reach an acceptable physical analytic solution for water depth in trapezoidal and triangular channels.

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1. Introduction

In many practical problems, alternate depths and sequent depths are determined by solving the specific energy and specific force equations, respectively. For a rectangular cross-section, the proportionality between the cross-sectional area and the depth leads to simple relationships between specific energy vs. depth and specific force vs. depth [1]. In this cross-section, inverting the specific energy and specific force functions corresponds to solving a cubic equation. Analytical solutions of the specific energy equation in rectangular channels are available in the technical literature [2,3]. Besides in [1] simple analytical solutions are presented for the inversion of the specific force vs. depth and specific energy vs. depth for wide rectangular cross-sections.

For other cross-sections such as trapezoidal or triangular, the specific energy and specific force equations yield quintic equations. Das developed a methodology for simultaneous determination of alternate depths and sequent depths in trapezoidal, rectangular and triangular cross-sections [4]. This iterative methodology uses solutions of quadratic and cubic equations to identify the two subcritical and supercritical depths. Vatankhah and Kouchakzadeh used iterative fixed-point method to present solutions of specific energy and specific force equations in open channels with trapezoidal, rectangular and triangular cross-sections [5]. They showed that distinguishing subcritical and supercritical depths using fixed-point method is possible. Valiani and Caleffi proposed an approximate analytical (not iterative) perturbation technique for solving specific energy and specific force equations in power law cross-sections [6]. The proposed method by Valiani and Caleffi is applicable for channels with power law cross-sections with exponent $m \leq 0.4$. The calculation steps of the method are not very straightforward and for inverting both specific energy and specific force requires that so many equations be engaged. Vatankhah proposed iterative fixed-point method for solution of specific energy and specific force equations in open channels with power law cross-sections [7].

In spite of a lot of investigations on the solutions of specific energy and specific force equations, from the above literature review it is revealed that there is an exact analytical solution only for rectangular channels. In current study, the specific energy and specific force equations in open channels with trapezoidal and triangular cross-sections are analytically inverted. In many situations, either subcritical or supercritical depth may be known along with channel geometry and discharge. For such cases, it is not necessary to determine the alternate depths or sequent depths simultaneously. In the current study analytical solutions to determine the other depth for such cases are presented. For this purpose governing equations are analytically inverted. Inversion of the specific energy and specific force equations involves the solution of quintic equations. Considering physically meaningful domains, this quintic equation is reduced to a quartic equation. This quartic equation can be solved using a classical method such as Ferrari’s solution [8–10], but further manipulations based on the algebraic analysis