Exact Sensitivity Equation for One-Dimensional Steady-State Shallow Water Flow (Application to Model Calibration)

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Abstract: Sensitivity analysis is an important tool that can be used for calibration of river flow models. Generally, friction coefficient is used as a calibration parameter for steady-state one-dimensional shallow water flow models. In this study, the local sensitivity analysis is used to derive a general rule for calibration of prismatic open channel models. For this, the sensitivity of the water depth to the friction coefficient is analytically derived for one-dimensional steady flow conditions in a wide rectangular channel. Also, the parameter of *influence distance* is defined to calibrate a channel reach using stage measurement at a given location. This characteristic distance allows 90% of the maximum possible variation in the sensitivity of the water depth to the friction coefficient to be captured. This parameter estimates the optimal length of a channel reach over which the friction coefficient can be calibrated from stage measurement. This length locates downstream of the stage measurement point in subcritical flow regime and upstream in supercritical flow regime.

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Introduction

A local sensitivity analysis is applicable to deterministic models. Local sensitivity analysis evaluates the effect on model outputs exerted by individually varying only one of the model inputs (Cullen and Frey 1999). The difference in the model output due to the change in the input variable is referred to as the sensitivity of the model to that particular input variable (Morgan and Henrion 1990). Hall et al. (2009) mentioned some reasons for using sensitivity analysis and reviewed a range of methods for sensitivity analysis in hydraulic models. As mentioned by the authors, sensitivity analysis can be used to find the optimal regions within the parameter space for use in calibration studies. In current research, the concept of local sensitivity is applied to continuous deterministic systems (such as shallow water flow models), which are described by differential equations.

Usually shallow water flow models are calibrated using steady-state assumptions (Guinot and Cappelaere 2009b). The friction coefficient is a key parameter in the calibration of river models (Guinot and Cappelaere 2009a). It seems that the first effort in deriving a general rule for one-dimensional river model calibration based on the sensitivity analysis should be attributed to Guinot and Cappelaere (2009a). The authors used empirical sensitivity, as well as approximate local perturbation approach, to establish *half-length* concept and presented a formula for the optimal definition of calibration reaches associated with stage measurement locations. They defined half-length as the distance after which the difference between the local value of depth sensitivity and its ultimate value is divided by 2. It should be noted that their formulation provides only an approximation of the half-length (region of influence) when the flow is nonuniform. Because of the importance of the local sensitivity analysis in river model calibration any effort to improve the application of this approach would be of practical importance.

Sensitivity equations may be derived by differentiating the model differential equation with respect to the desired parameter or solving the governing differential equation, then differentiating the resulting algebraic solution with respect to the desired parameter. The latter approach can be used when an analytical solution is available for the governing differential equation. In current research, the latter approach was applied, by deriving an analytical solution for the governing differential equation in wide rectangular channels. Then an analytical solution was obtained for depth sensitivity. This solution can be used to determine the influence distance under steady-state conditions. The influence distance allows 90% of the maximum possible variation in the sensitivity of the water depth to the friction coefficient to be captured. Note that this definition has a precise physical meaning. This distance expresses a length over which the water depth is sensitive to a variation in the friction coefficient. Beyond the influence distance, the water depth is assumed insensitive to a variation in the friction coefficient. Using this definition the optimal length of a river/channel reach for adjusting the friction coefficient based on depth measurement at a given location can be determined.

It should be noted that we used second approach for a wide rectangular channel. This approach provides clear understanding of the influence distance concept for this simple section. For other

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