Discussion of "Head-Discharge Equation for Sharp-Crested Polynomial Weir" by Raouf E. Baddour

April 2008, Vol. 134, No. 2, pp. 260–262.
DOI: 10.1061/(ASCE)0733-9437(2008)134:2(260)

Ali R. Vatankhah[1] and Salah Kouchakzadeh[2]
[1]Former Ph.D. Student, Irrigation and Reclamation Engineering Dept., Univ. College of Agriculture and Natural Resources, Univ. of Tehran, Iran. E-mail: alireza_vatankhah@yahoo.com
[2]Prof., Irrigation and Reclamation Engineering Dept., Univ. College of Agriculture and Natural Resources, Univ. of Tehran, Iran. E-mail: skzadeh@ut.ac.ir

The discussers would like to thank the author for presenting a head-discharge equation for a sharp-crested polynomial weir. As stated by the author, a polynomial of an order higher than 4 is required to provide a better fit to a Sutro weir; therefore, it would be more convenient to present Eq. (7) in the original paper in a simpler form, which could be attained by using the Gamma function. The general form of the discharge relation for polynomial weir of nth order takes the form

$$Q = K\int_0^H \sqrt{H-y}(a_0 + a_1 y + a_2 y^2 + \ldots + a_n y^n)dy \qquad (1)$$

where $x=y/H$, and reduces Eq. (1) to

$$Q = \sum_{i=0}^{n} Q_i \qquad (2)$$

in which

$$Q_i = K a_i H^{i+3/2}\int_0^1 x^i\sqrt{1-x}dx \qquad (3)$$

The beta function could be used to evaluate Eq. (3). The beta function and its relation to the Gamma function is presented by Eq. (4) (Spiegel 1999)

$$\int_0^1 x^{m-1}(1-x)^{n-1}dx = \frac{\Gamma[m]\Gamma[n]}{\Gamma[m+n]} \qquad (4)$$

where n and $m>0$ are beta function parameters and Γ is a gamma function. Using Eq. (4), the integral term in Eq. (3) could be evaluated as follows:

$$\int_0^1 x^i\sqrt{1-x}dx = \frac{\Gamma[i+1]\Gamma[3/2]}{\Gamma[i+5/2]} \qquad (5)$$

which for integer $i\geqslant 0$ yields

$$\int_0^1 x^i\sqrt{1-x}dx = \frac{2^{i+1}i!}{1\times 3\times 5\times \ldots \times(2i+3)}$$

Substituting the above equation in Eq. (3) yields

$$Q = K\sum_{i=0}^{n} a_i H^{i+3/2}\frac{2^{i+1}i!}{1\times 3\times 5\times \ldots \times(2i+3)} \qquad (6)$$

Eq. (6) need not be a polynomial transformation in which the determination of α_i is required; therefore, it presents a simpler form than Eq. (7) from the original paper.

It should also be noted that in using Eq. (7) instead of Eq. (2) of the original paper another form of Eq. (6) could be developed.

$$b = a_0 + \sum_{i=1}^{n} a_i y^{(2i-1)/2} \qquad (7)$$

Based on Eq. (7) the discharge relation of weir becomes

$$Q = \frac{2}{3}Ka_0H^{3/2} + K\int_0^H \sqrt{H-y}(a_1y^{1/2} + a_2y^{3/2} + \ldots + a_ny^{(2n-1)/2})dy \qquad (8)$$

Considering $x=y/H$, Eq. (8) takes the form

$$Q = \frac{2}{3}Ka_0H^{3/2} + \sum_{i=1}^{n} Q_i \qquad (9)$$

in which

$$Q_i = Ka_iH^{i+1}\int_0^1 x^{i-1/2}\sqrt{1-x}dx \qquad (10)$$

Again based on Eq. (4) the integral term in Eq. (10) could be evaluated as follows:

$$\int_0^1 x^{i-1/2}\sqrt{1-x}dx = \frac{\Gamma[i+1/2]\Gamma[3/2]}{\Gamma[i+2]}$$

which for integer $i\geqslant 1$, yields

$$\int_0^1 x^{i-1/2}\sqrt{1-x}dx = 1\times 3\times 5\times \ldots \times(2i-1)\frac{\pi}{2^{i+1}(i+1)!}$$

and hence

$$Q = \frac{2}{3}Ka_0H^{3/2} + \pi K\sum_{i=1}^{n} a_iH^{i+1}\frac{1\times 3\times 5\times \ldots \times(2i-1)}{2^{i+1}(i+1)!} \qquad (11)$$

For $n=4$ Eq. (11) reduces to

$$Q = K\left(\frac{2}{3}a_0H^{3/2} + \frac{\pi}{8}a_1H^2 + \frac{\pi}{16}a_2H^3 + \frac{5\pi}{128}a_3H^4 + \frac{7\pi}{256}a_4H^5\right) \qquad (12)$$

In such a case, the head-discharge relation (instead of b) is polynomial except for first term.

Exact Solution of Proportional Weir

In regard to Eq. (3), for proportional or Sutro weir i should be $-1/2$. Therefore, the discharge is