The discussers appreciate the author’s presentation of a new methodology for simultaneous determination of alternate and sequent depths. The traditional fixed-point method, however, could be used to present solutions that overcome the difficulty of the traditional computational procedures and converge faster toward the solution than the method proposed by the author. The dimensionless specific energy equation for a trapezoidal cross section takes the form

\[ f_E(y_*) = E_* - y_* - \frac{Q_*}{(1 + y_*)^2}y_* = 0 \]  

(1)

where \( y_* = Zy/B, E_* = ZE/B \) and \( Q_* = \frac{Q^2}{(2gB^3)} \).

Different iterative arrangements of Eq. (1) in the form of \( y_{n+1} = g(y_n), n = 0, 1, 2, 3, \ldots \) might be considered for alternate depths determination. However, the most suitable form that converges to the desired root by using the fixed-point iteration technique should be sought.

Two arrangements of \( f_E(y_*) \) in the forms of Eqs. (2) and (3) are proposed herein for calculating the nondimensional alternate depths \( y_{s2} = Zy_2/B \) and \( y_{s1} = Zy_1/B \), respectively. The subscripts 1 and 2 refer to the small and the large depths, respectively

\[ y_* = g_2(y_*) = E_* - \frac{Q_*}{(1 + y_*)^2}y_*^2 \]  

(2)

\[ y_* = g_1(y_*) = -0.5 + \sqrt{0.25 + \frac{Q_*}{E_* - y_*}} \]  

(3)

The general forms of \( g_2(y_*) \) and \( g_1(y_*) \) are drawn in Fig. 1. The variable \( g_2(y_*) \) has two asymptotes (i.e., \( z_0 = E_* \) and \( z_1 = 0 \)) and intersects with the abscissa at \( y_{s2} = -0.5 + \{0.25 + (Q_*/E_*)^{0.5}\}^{0.5} \). Also, \( g_1(y_*) \) has two asymptotes (i.e., \( z_0 = 0 \) and \( z_1 = E_* \)) and intersects with the ordinate at \( y_{s2} = -0.5 + \{0.25 + (Q_*/E_*)^{0.5}\}^{0.5} \).

The slope of \( g_1(y_*) \) and \( g_2(y_*) \) functions near the root governs their behavior. That is, starting the computation from points having a slope close to zero tend to increase the convergence rate, while using any point with slope greater than +1 would either diverge the iteration process or produce another root. Considering 0.1\( E_* \) (10% of the asymptote value) as an initial guess for calculating \( y_{s1} \) and 0.9\( E_* \) (90% of the asymptote value) for calculating \( y_{s2} \) speed up the convergence process.

For a rectangular cross section, Eq. (1) simplifies to

\[ f_{ER}(y_*) = 1 - y_* = \frac{Q_*/R}{y_*^2} = 0 \]  

(4)

where \( y_*/R = y/E \) and \( Q_*/R = Q^2/(2gB^2E^3) \).

An appropriate arrangement of \( g_{ER}(y_*) \) for calculating \( y_{s2} = y_2/E \) and \( y_{s1} = y_1/E \) respectively could take the following forms

\[ y_* = g_{ER}(y_*) = 1 - \frac{Q_*/R}{y_*^2} \]  

(5)

\[ y_* = g_{1R}(y_*) = \sqrt{\frac{Q_*/R}{1 - y_*^4}} \]  

(6)

Using 90% and 10% of the vertical asymptote value of Eq. (6) (i.e., \( y_*/R = 1 \)) as initial guesses would guarantee fast convergence to the desired roots \( y_{s2} \) and \( y_{s1} \), respectively.

Similarly for a triangular cross section Eq. (1) simplifies to

\[ f_{ET}(y_*) = 1 - y_* = \frac{Q_*/T}{y_*^4} = 0 \]  

(7)

where \( y_*/T = y/E \) and \( Q_*/T = Q^2/(2gZ^3E^3) \). For a triangular cross section appropriate arrangements for \( g_{ET}(y_*) \) and \( g_{1T}(y_*) \) for calculating \( y_{s2} = y_2/E \) and \( y_{s1} = y_1/E \) respectively could be presented in the following forms:

\[ y_* = g_{ET}(y_*) = 1 - \frac{Q_*/T}{y_*^4} \]  

(8)

\[ y_* = g_{1T}(y_*) = 4 \left( \frac{Q_*/T}{1 - y_*^4} \right)^{0.25} \]  

(9)

For calculating the roots of Eqs. (8) and (9) (i.e., \( y_{s2} \) and \( y_{s1} \)) the condition for initial guesses as previously described might be used for the triangular cross section.

The same procedure was pursued for solving the dimensionless specific force equation for a trapezoidal cross section [Eq. (10)]