
The discusser would like to thank the authors for using an approximate analytical (not iterative) perturbation technique for the power-law cross sections. The work by the authors in accomplishing an explicit perturbation solution applicable to power-law cross sections is really appreciated. However, the discusser would like to introduce an easily implementable iterative procedure with exemplified convergence behaviour.

The proposed method by the authors is applicable for channels with power-law cross sections with exponent \( m \) \( \leq 0.4 \). To be used for all encountered channels of power-law cross section the procedure should also be developed for \( m \) values between 0.4 and 1. However the calculation steps of the method are not as straightforward as mentioned by the authors, for inverting both non-dimensional specific energy and total force requires that so many equations be engaged.

A fixed-point iterative method is introduced herein which circumvents tedious computational steps and shown to have a rapid convergence toward the final solution. Also, it makes distinguishing subcritical and supercritical depths possible. Vatankhah and Kouchakzadeh [1] used this method for solution of specific energy and specific force equations in open channels with trapezoidal, rectangular and triangular cross sections. Using this iterative method, solution of specific energy and specific force equations is possible for power-law cross-sections. The fixed-point method covers entire practical range of \( 0 \leq m \leq 1 \).

1. Solution of non-dimensional specific energy equation

   Considering \( \alpha = 2 + 2m \) and \( \Gamma^* = \Gamma(3 + 2m) \), Eq. (10) of the original paper takes the form:
   \[
   \Gamma^* = \alpha \eta + \frac{1}{\eta^2}
   \]
   In which \( 2 \leq \alpha \leq 4 \), Eq. (1) can be written as:
   \[
   f_3(\eta) = \Gamma^* - \alpha \eta - \frac{1}{\eta^2}
   \]  

   The alternate depths can be obtained by solving Eq. (2) for a given \( \Gamma^* \) and \( \alpha \). Different iterative arrangements of Eq. (2) in the form of \( \eta_{n+1} = g(\eta_n) \), \( n = 0, 1, 2, 3 \ldots \) might be considered for determination of alternate depths. However, the most suitable form to be used in fixed-point iteration technique that converges to the desired root should be sought. Two arrangements of Eq. (2) in the forms of Eqs. (3) and (4) are proposed herein for calculating the non-dimensional alternate depths \( \eta_1 \) and \( \eta_2 \), respectively. The subscripts 1 and 2 refer to the supercritical and subcritical depths, respectively

   \[
   \eta = g_2(\eta) = \frac{1}{2} \left( \Gamma^* - \frac{1}{\eta^2} \right)
   \]  

   \[
   \eta = g_1(\eta) = (\Gamma^* - \alpha \eta)^{-1/2}
   \]

   The general forms of \( g_2(\eta) \) and \( g_1(\eta) \) are drawn in Fig. 1. The variable \( g_2(\eta) \) has two asymptotes (i.e., \( z = \Gamma^*/\alpha \) and \( \eta = 0 \)) and it intersects the abscissa at \( \eta = \Gamma^*/\alpha \). Also, \( g_1(\eta) \) has two asymptotes (i.e., \( z = 0 \) and \( \eta = \Gamma^*/\alpha \)) and intersects the ordinate at \( z_e = \Gamma^*/\alpha \).

   The slope of \( g_2(\eta) \) and \( g_1(\eta) \) functions near the root governs their behaviour. That is, starting the computation from points having a slope close to zero tends to increase the convergence rate, while using any point with slope greater than +1 would either diverge the iteration process or produce another root. Considering \( 0.2 \Gamma^*/\alpha \) (20% of the asymptote value) as an initial guess for calculating \( \eta_1 \) and \( 0.9 \Gamma^*/\alpha \) (90% of the asymptote value) for calculating \( \eta_2 \) speed up the convergence process.

2. Solution of non-dimensional specific force equation

   The same procedure was pursued for solving the non-dimensional specific force equation for a power-law cross section. Considering \( \beta = \alpha/2 = 1 + m \) and \( \Phi^* = \Phi(3 + 2m) \), Eq. (17) of the original paper can be written as:

   \[
   \Phi^* = \beta \eta^{1-\beta} + (1 + \beta) \frac{1}{\eta^2}
   \]

   In which \( 1 \leq \beta \leq 2 \), Eq. (5) can be rearranged as:

   \[
   f_4(\eta) = \Phi^* - \beta \eta^{1-\beta} - (1 + \beta) \frac{1}{\eta^2}
   \]

   The sequent depths are obtained by solving Eq. (6). An appropriate arrangement of \( h_2(\eta) \) and \( h_1(\eta) \) for determination of \( \eta_2 \) and \( \eta_1 \) is given by Eqs. (7) and (8), respectively

   \[
   \eta = h_2(\eta) = \left( \frac{\Phi^*}{\beta} - \frac{1 + \beta}{\beta \eta} \right)^{1/(1+\beta)}
   \]  

   \[
   \eta = h_1(\eta) = \left( \frac{1 + \beta}{\Phi^* - \beta \eta^{1-\beta}} \right)^{1/\beta}
   \]

   Fig. 2 presents the general form of \( h_2(\eta) \) and \( h_1(\eta) \) functions. The function \( h_1(\eta) \) has two asymptotes (i.e., \( z = (\Phi^*/\beta)^{1/(1+\beta)} \) and \( \eta = 0 \)) and it intersects the abscissa at \( \eta = [(1 + \beta)/\Phi^*]^{1/\beta} \). Likewise,

**DOI of original article: 10.1016/j.advwatres.2008.10.015**

0309-1708/$ - see front matter © 2009 Elsevier Ltd. All rights reserved. doi:10.1016/j.advwatres.2009.02.008