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# Direct integration of Manning-based gradually varied flow equation

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**Solution of the gradually varied flow equation allows tracing of the flow depth along a channel length. Direct solutions of this differential dynamic equation are available in the technical literature only for the special case of wide rectangular channels and for general rectangular channels based on the simple Chezy formula. No direct solution is available for the case of general rectangular channels based on the complex, but more realistic, Manning formula. This paper presents an approach for establishing the gradually varied flow profile for general rectangular channels through application of the Manning formula. The approach, which yields a very accurate computation of flow profiles, should be a useful tool for direct quantitative analysis and evaluations of general rectangular channels.**

## Notation

| | |
|---|---|
| $A$ | cross-sectional area of flow (m$^2$) |
| $B$ | bottom width of the channel (m) |
| $C$ | Chezy constant |
| $dy/dx$ | slope of the free surface at any location |
| $g$ | gravitational acceleration (m/s$^2$) |
| $K$ | dimensionless constant |
| $n$ | Manning's roughness coefficient |
| $P$ | wetted perimeter (m) |
| $Q$ | water discharge (m$^3$/s) |
| $r_0$ | dimensionless constant |
| $S_0$ | longitudinal slope of the channel bottom |
| $T$ | width of water surface (m) |
| $x$ | distance along the channel, considered positive in the downstream direction (m) |
| $y$ | depth of flow (m) |
| $y_0$ | normal depth of flow (m) |
| $y_c$ | critical depth of flow (m) |
| $z$ | water elevation (m) |
| $\delta$ | function of channel geometric and flow parameters |
| $\varepsilon$ | dimensionless constant |
| $\eta$ | dimensionless depth of flow ($y/B$) |
| $\eta_c$ | dimensionless critical depth of flow ($y_c/B$) |
| $\eta_1$ | dimensionless depth of flow $y_1$ ($y_1/B$) |
| $\eta_2$ | dimensionless depth of flow $y_2$ ($y_2/B$) |
| $\chi$ | dimensionless distance along the channel |

## 1. Introduction

The computation of gradually varied flow (GVF) profiles is of considerable importance to hydraulic engineers (Chow, 1959). In many practical problems, an accurate procedure for determining water elevation in various sections of a channel is required. This procedure can be carried out either numerically or analytically. Numerical integration of the GVF dynamic equation is mainly used in non-prismatic channels. In some prismatic channels, the governing equation is simplified and direct (analytical) integration can be applied. In such cases, the integration is straightforward and the total length of the profile can be calculated in a single computational step. However, such integration methods do not directly provide the depth of flow at a specific distance along the channel. A summary of existing solutions of the GVF equation that can be used to establish flow profiles is shown in Table 1.

Bresse (1868) derived an analytical solution for a wide rectangular channel using the Chezy formula. His results were based on the Chezy coefficient ($C$), which is constant in the Chezy formula; in reality, however, $C$ depends on the flow characteristics (e.g. hydraulic radius and channel bed slope). Bakhmeteff (1932) proposed a direct integration applicable to any channel shape. His method is an approximate integration method that requires dividing the channel length into short reaches. Chow (1955) developed an extension of Bakhmeteff's method and eliminated its computational complexity. Gunder (1943) integrated the GVF