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Analytical solutions for Bingham plastic fluids in laminar regime

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ABSTRACT

An analytical approach is used to develop true solutions for calculating the friction factor, and pipe diameter under laminar flow conditions of Bingham plastic fluids in pipe networks. For this, governing equations have been converted to suitable quartic equations. In the next step, each of the quartic equations has been converted to a resolvent cubic equation and two quadratic equations. This research shows these steps clearly to reach acceptable physical analytic solutions for calculating the friction factor and pipe diameter.
Proposed true analytical solutions aim at reducing the computational time and effort and eliminate the need for iterative solutions. The propagation of errors for nonlinear systems should be accounted for when the intensive computation of the friction factor (e.g. in optimization problems) is required. Proposed solution of the friction factor will be a suitable computational tool for implementing these nonlinear models.

1. Introduction

Non-Newtonian fluids have many applications in geophysics, petroleum engineering, meteorology, turbomachinery and many other fields. Many industrially non-Newtonian fluids, such as various suspensions, slurries, pastes, gels and plastics are represented by Bingham fluids (Bird et al., 2002). The Bingham plastic model is one of the practical models in the design and analysis of pipelines for conveying non-Newtonian fluids (Sablani et al., 2003). The hydraulic analysis and design of non-Newtonian pipeline networks depend upon predicting the friction factor and pipe diameter. Indeed, unknown friction factor (analysis problem) and unknown diameter (design problem) problems are two typical problems encountered in pipe flows of Non-Newtonian fluids. Determining the solution to these flow problems is difficult due to the implicit nature of the governing equations.

No analytical dimensionless solutions are available in the technical literature for these two typical problems. Exact analytical solutions of the governing equations can be obtained because they are fourth order polynomial. Up till now, it is assumed that it is extremely difficult to derive an exact analytical solution to calculate the friction factor for flow of non-Newtonian fluids under laminar flow conditions and therefore explicit approximations are used (Swamee and Aggarwal, 2011). As it will be shown in next sections, the solution of the governing equations consists of finding the roots of quartic equations. A quartic equation can be solved using a classical method such as Ferrari's solution (Abramowitz and Stegun, 1972; Beyer, 1987), but further manipulations and simplifications based on the algebraic analysis and physical meaning of some terms make the final result much more friendly to use for design engineers.

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2. Governing equations

The friction factor, f, for the laminar flow of Bingham plastic fluids can be written in terms of the Bingham Reynolds number, Re, and the Hedstrom number, He, using the Buckingham–Reiner equation as (Darby and Melson, 1981; Govier and Aziz, 1972)

$$f=\frac{64}{\mathrm{Re}}\left(1+\frac{\mathrm{He}}{6\mathrm{Re}}-\frac{64\,\mathrm{He}^4}{3f^3\mathrm{Re}^7}\right). \tag{1}$$

The Reynolds number and the Hedstrom number were respectively defined as

$$\mathrm{Re}=\frac{VD}{\nu_\infty} \tag{2}$$

$$\mathrm{He}=\frac{\tau_0 D^2}{\rho\nu_\infty^2} \tag{3}$$

in which ρ is the mass density of fluid, ν_∞ is the kinematic viscosity of Bingham fluid, D is the pipe diameter, V is the mean flow velocity and τ_0 is the yield shear stress of fluid.

Eq. (1) can be written in terms of two variables (ε and f_*) as follows:

$$f_*^4-\varepsilon f_*^3+\frac{1}{3}=0 \tag{4}$$

in which

$$f_*=\frac{f\mathrm{Re}^2}{8\mathrm{He}} \tag{5}$$