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Discussion: Hydraulic jumps in trapezoidal and circular channels

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The author has determined the vertical distance between the centroid of the flow area and the water surface, by equating the two areas above and below the horizontal centroid line. This derivation is not generally true. It is correct only if the horizontal centroid line coincides with the horizontal symmetric axis of the section. The author also derived Equations 4a and 4b with respect to his definition incorrectly.

The centroid position could be determined by taking moments about the invert axis of the section as follows

$$A(y-\bar{z}) = \int y\,\mathrm{d}A \tag{16}$$

Rearranging Equation 16 yields

$$A\bar{z} = Ay - \int y\,\mathrm{d}A = \int \mathrm{d}(Ay) - \int y\,\mathrm{d}A \tag{17}$$

Equation 17 can be simplified as

$$A\bar{z} = \int y\,\mathrm{d}A + \int A\,\mathrm{d}y - \int y\,\mathrm{d}A = \int A\,\mathrm{d}y \tag{18}$$

Centroid position for a trapezoidal section

Centroid position for a trapezoidal section can be determined as follows

$$A\bar{z} = \int (B+sy)y\,\mathrm{d}y = By^2/2 + sy^3/3 \tag{19}$$

By differentiation of the left-hand side of Equation 19 with respect to y, the following is obtained

$$\frac{\mathrm{d}(A\bar{z})}{\mathrm{d}y} = By + sy^2 = A \tag{20}$$

Equation 20 can be used for the Newton–Raphson method.

Centroid position for a circular section

Centroid position for a circular section can be determined as follows

$$A\bar{z} = \int A\,\mathrm{d}y = \frac{D^3}{8}\int(\theta - \sin\theta)\,\mathrm{d}\eta \tag{21}$$

in which

$$\theta = 2\cos^{-1}(1-2\eta) \tag{22}$$

where θ is the water surface angle in radians according to Figure 3, and $\eta = y/D$ and y denotes the depth of the water in the pipe.

Substituting Equation 22 into Equation 21 and integrating yields

$$A\bar{z} = D\left(\eta - \frac{1}{2}\right)A + \frac{2}{3}D^3(\eta - \eta^2)^{3/2} \tag{23}$$

By differentiating the left-hand side of Equation 23 with respect to y, the following is obtained

$$\begin{aligned}\frac{\mathrm{d}(A\bar{z})}{\mathrm{d}y} &= \frac{1}{D}\frac{\mathrm{d}(A\bar{z})}{\mathrm{d}\eta}\\ &= D^3\frac{\mathrm{d}}{\mathrm{d}y}\left[\left(\eta-\frac{1}{2}\right)\frac{A}{D^2} + \frac{2}{3}(\eta-\eta^2)^{3/2}\right] = A\end{aligned} \tag{24}$$

In general using Equation 18 it is proved that

$$\frac{\mathrm{d}(A\bar{z})}{\mathrm{d}y} = \frac{\mathrm{d}}{\mathrm{d}y}\int A\,\mathrm{d}y = A \tag{25}$$

Equation 25 can be used for the Newton–Raphson method; however, this method is not the most efficient for calculating conjugate depths. The fixed-point method could also be used to