Briefing: Non-iterative solution for positive surge waves

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The propagation of positive surge waves in rectangular open channels is important in practice. For this phenomenon, the governing equations are derived using the continuity and momentum equations. The existing solutions of these equations are based on trial or graphical procedures. No direct analytical solution is available in the technical literature for this phenomenon even for the case of geometrically simple cross-sections. This paper presents such a solution.

The general theory of positive surge waves has been described by Chow (1959) and French (1994). Specific descriptions of open-channel positive surge have been presented by several researchers, including Haws (1954), Press and Schroeder (1966), Dey (1989), Yang et al. (1992) and Hsu and Yeh (2002). A graphical solution of the positive surge wave problem has been presented for rectangular, trapezoidal and parabolic channel cross-sections by Istemi (1979). More recently, Mitchell (2010) presented an iterative analytical solution for rectangular, trapezoidal and circular channels and also gave a review of early experiments related to the positive surge wave phenomenon. However, despite this extensive research, no direct analytical solution for this phenomenon has yet been reported.

The purpose of this article is to present a non-iterative analytic solution for positive surge waves in rectangular open channels. The idea is to eliminate the surge wave velocity from the governing equations, which results in a quartic equation for the flow depth ratio. The inversion of such equations consists of finding the roots of quartic equations. By solving the governing equations, two water depth ratios for positive surge waves moving in the upstream and downstream directions are found analytically.

In this case, two acceptable roots are adopted and the two other roots are discarded on physical grounds. This research shows that these steps clearly lead to a physically meaningful solution for the water depth ratio. Such a solution can be used in engineering practice and can help hydraulic students better understand the surge phenomenon in open channels.

1. Introduction
The propagation of positive surge waves is of practical interest in diversion and navigation channels, water and wastewater treatment works, the design of irrigation channels and hydroelectric power stations. Surges occur when there is a sudden change in flow, caused by, for example, closure of a downstream gate, opening of an upstream gate or a dam break, which increases water depth (Chanson, 2004) (Figure 1). A positive surge wave moving in the upstream direction of flow is called an upstream positive surge and a positive surge wave moving in the downstream direction of flow is called a downstream positive surge. The strength of a surge depends mainly on the change in the flow. When the change is very small, the surge is undular, like a surface wave. However, when the change in flow is large, a stronger surge develops. It is a rapidly varying unsteady flow condition that may be described using the basic principles of conservation of mass and momentum.

The general theory of positive surge waves has been described by

Notation

- $g$: gravitational acceleration
- $Q_1$: initial flow discharge
- $Q_2$: new flow discharge
- $V_1$: undisturbed flow velocity
- $V_2$: disturbed flow velocity
- $V_{swd}$: absolute velocity of the downstream surge wave
- $V_{swu}$: absolute velocity of the upstream surge wave
- $y_1$: initial flow depth
- $y_2$: new flow depth
- $\varepsilon$: discharge ratio ($Q_2/Q_1$)
- $\eta$: flow depth ratio ($y_2/y_1$)