Direct Integration of Manning-Based GVF Equation in Trapezoidal Channels

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Abstract: The direct integration method is used to compute free surface profiles in gradually varied flow (GVF) along the length of a prismatic open channel. Analytical solutions of the GVF equation, based on the Manning equation, are available in the technical literature only for the special case of triangular and wide rectangular channels. No closed-form (direct) solution is available for this equation for the case of trapezoidal channels. Open channels with trapezoidal cross sections are widely used as drainage, irrigation, urban stormwater, water transmission, and power channels. In the current study, by applying the Manning equation, a semianalytical solution to compute the length of the GVF profile for trapezoidal channels is derived. This solution, which allows an accurate computation of the flow profiles with minimal computational effort and time, should be a useful tool for direct quantitative analysis and evaluations of trapezoidal channels and thus should be of interest to practitioners in the water engineering community. **DOI: 10.1061/(ASCE)HE.1943-5584.0000460.** © *2012 American Society of Civil Engineers.*

CE Database subject headings: Gradually varied flow; Open channels; Integrated systems.

Author keywords: Gradually varied flow profiles; Direct integration; Manning equation; Trapezoidal channels.

Introduction

Sketching of water surface profiles in open channels with gradually varied flow (GVF) has been discussed in many textbooks. However, the computations of the GVF profiles are of considerable importance to hydraulic and hydrologic engineers. The most widely used methods for computing the flow profiles are classified into step methods and direct integration methods. The step methods (Chow 1959) are numerical in nature and can be used when two flow depths are given and the distance between them is required (direct step method) or when the flow depth at a specified location is required (standard step method). The direct integration methods involve the integration of the GVF governing equation and may be performed using analytical, semianalytical, or numerical procedures. Numerical integration of the GVF dynamic equation is primarily used in nonprismatic channels. In some prismatic channels, the governing equation is simplified and analytical or semianalytical integration can be applied. In such cases, the integration is straightforward and the total length of the profile can be calculated in a single computational step. However, such integration methods do not directly provide the depth of flow at a specific distance along the channel length. The concentration of this research is on direct integration methods.

Bresse (1868) derived an analytical solution for wide rectangular channels using the Chézy equation, but the roughness coefficient was considered constant. In general, however, the roughness coefficient (C-coefficient) depends on hydraulic radius and channel bed slope. Bakhmeteff (1932) proposed a direct integration that is applicable to any channel shape. His approximate integration method requires dividing the channel length into short reaches. Gunder (1943) integrated the GVF equation of a wide rectangular channel considering the variation of the Chézy coefficient with depth. Chow (1955) developed an extension of Bakhmeteff's method that eliminated its computational complexity. Gill (1976) derived an analytical solution for a wide rectangular channel using the Manning equation. The solutions presented by Gunder (1943) and Gill (1976) are preferable to Bresse's solution for wide rectangular channels. Gill (1976) also derived some exact solutions for channels with different constant values of the hydraulic exponent. A direct integration for general rectangular and triangular channels has also been proposed by Kumar (1978) using the Chézy equation, but the Chézy coefficient was considered constant. Dubin (1999) proposed an approximate semianalytical solution for a rectangular channel using the Manning equation. Ramamurthy et al. (2000) integrated the series expansion of the dynamic equation for GVF flows. However, a large number of terms should be used for obtaining accurate results. Vatankhah (2010a) proposed an analytical solution for triangular channels using the Manning equation. Venutelli (2004) and Vatankhah (2010b) presented analytical solutions for a wide rectangular channel using the Manning equation. Vatankhah and Easa (2011) derived a semianalytical solution for general rectangular channels using the Manning equation. Vatankhah (2011) also derived a semianalytical solution for parabolic channels.

From the preceding literature review, it is clear that there is no direct (semianalytical) solution to compute the GVF profiles for trapezoidal channels. Trapezoidal open channels are used in many hydrology and hydraulic applications (Chow 1959; Chaudhry 2006; Haltas and Kavvas 2009; Wong and Zhou 2006). Open channels with trapezoidal cross section are widely used as urban stormwater, drainage, irrigation, water transmission, and power channels (Das 2007). In this study, the direct integration is used to determine the flow profiles in a trapezoidal channel based on the Manning equation. The following sections present the governing equation

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Note. This manuscript was submitted on October 19, 2010; approved on June 10, 2011; published online on June 14, 2011. Discussion period open until August 1, 2012; separate discussions must be submitted for individual papers. This technical note is part of the ***Journal of Hydrologic Engineering***, Vol. 17, No. 3, March 1, 2012. ©ASCE, ISSN 1084-0699/2012/3-455–462/$25.00.