

Short communication

# Improved explicit approximation of linear dispersion relationship for gravity waves: A discussion

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## ABSTRACT

Recently, an accurate explicit approximation to linear dispersion relationship is proposed based on Eckart's explicit relationship (Beji, 2013). The author has nicely improved Eckart's explicit dispersion relationship by introducing an empirical correction function. The resulting expression is valid for the entire range of relative water depths and accurate to within 0.044%.

In this discussion, the proposed expression by the author is simplified and improved to an accuracy of 0.019%. Moreover, a near exact solution with 0.001% accuracy is also given.

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## 1. Introduction

The discussers thank the author for nicely improving Eckart's explicit dispersion relationship. The discussers, however, would like to add a few points.

In the original paper for determining the coefficient of Eq. (5), the exact dispersion relationship for each  $\mu_0$  is obtained by applying the midpoint formula to successive iterations until the absolute value of the difference between two successive values is less than  $10^{-6}$ . However, this procedure can be easily conducted, and the iterative method is unnecessary. For this,  $\mu_0$  is first computed for a given accurate  $\mu$ , and then estimated  $\mu$  is computed from estimation expression (5) using computed  $\mu_0$  in terms of  $\mu$ . Then estimated  $\mu$  by expression (5) and  $\mu$  can be used for error analysis.

To further develop an improved explicit solution in the current study, Eq. (5) of the original paper is first modified using the curve fitting method as follows

$$\mu = \frac{\mu_0 + \mu_0^a e^{-(b+c\mu_0^d)}}{\sqrt{\tanh \mu_0}} \quad (1)$$

where  $a$ ,  $b$ ,  $c$  and  $d$  are coefficients. To determine these coefficients, the percentage error (PE) of the dispersion parameter,  $\mu$ , is expressed as follows

$$PE = \frac{\mu - \frac{\mu_0 + \mu_0^a e^{-(b+c\mu_0^d)}}{\sqrt{\tanh \mu_0}}}{\mu} \times 100 \quad (2)$$

in which  $\mu_0 = \mu \tanh \mu$ . Then, the maximum PE values, that is, Max |PE| is minimized as an objective function using the Solver toolbox of Microsoft Excel. The resulting explicit approximation in the practical range of  $\mu \in [0, \infty]$  is given by

$$\mu = \frac{\mu_0 + \mu_0^2 e^{-(1.835+1.225\mu_0^{1.35})}}{\sqrt{\tanh \mu_0}} \quad (3)$$

The maximum error involved in Eq. (3) is less than 0.019% in the practical range of  $\mu \in [0, \infty]$ . As noted in terms of accuracy (and simplicity), the discussers' new approximation (Eq. (3)) has advantage over the author's solution (Eq. (5)).

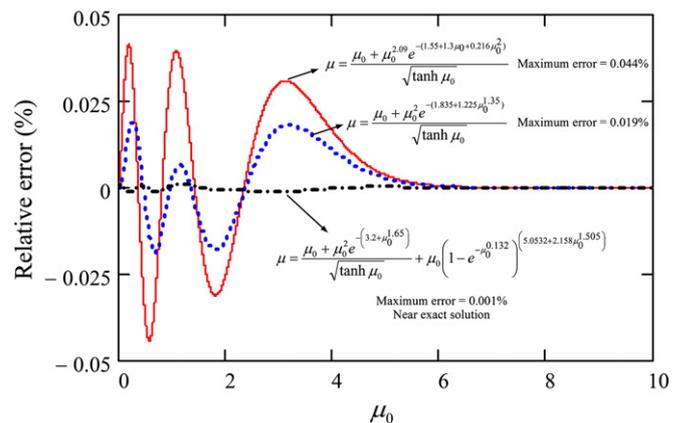


Fig. 1. Relative errors associated with Eqs. (3) and (4) presented in this discussion and one presented by the author (Eq. (5)).

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