Technical Note

Multiple Critical Depth Occurrence in Two-Stage Cross Sections: Effect of Side Slope Change

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Abstract: The critical flow concept has many applications in open-channel flow problems. Determination of critical depths for canals and natural streams with different shapes is a classic task, but important for efficient hydraulic design. Generally, critical depth is used to classify the flow into subcritical and supercritical and may be used as a control point for computing water surface profiles in steady and unsteady gradually varied flows. For a given compound channel, the kinetic energy correction coefficient varies quite rapidly above the overbank level. In such a case, specific energy may have more than local minimum or maximum (multiple critical depth) for some combination of discharge and geometry. The compound channels are well documented in the literature, but occurrence condition of the multiple critical depth has not yet been determined. The first step in modeling critical depth is to determine whether multiple occurrences exist. In current research, symmetrical prismatic channels with two different side slopes (two-stage cross sections) are considered, and the condition for which multiple critical depths exist is mathematically determined. For this, an analytical procedure is used to determine limiting side slope for the upper portion of the cross section. If the side slope of the upper portion is milder than the limiting side slope, the multiple critical depths will occur in the two-stage cross section. **DOI: 10.1061/(ASCE)HE.1943-5584.0000682.**

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Introduction

Critical flow depth is determined by setting the first derivative of specific energy with respect to depth equal to zero (Chow 1959). Critical flow regime is unstable, and a small fluctuation in energy will shift the flow into subcritical or supercritical. For simple cross sections, there is only one critical depth for a particular discharge. Prediction of the critical flow depth in a simple prismatic channel can be achieved to a high degree of accuracy using one-dimensional flow modeling. One-dimensional flow modeling is also acceptable for river channel. In compound open channels, strong lateral flow occurs when water is submerging the floodplains. For these channels, multiple critical depths can be observed for some cross sections.

The source of the multiple critical depths is clearly the sudden change in top water width as the flow leaves the main channel. Many researchers have confirmed the existence of more than one critical depth in compound open channels (Petryk and Grant 1978; Blalock and Sturm 1981; Schoellhamer et al. 1985; Chaudhry and Bhallamudi 1988; Lee et al. 2002). Proper identification of the existence of multiple critical depths is necessary in gradually varied flow computations of the compound channels. For the compound channels, kinetic energy correction factor varies quite rapidly above the overbank level (Subramanya 2009). In such a case, specific energy may have more than local minimum or maximum (multiple critical depths) for some combination of discharge and geometry (Subramanya 2009). Blalock and Sturm (1981) also observed experimentally two minima in the specific-energy curve. The compound channels are well studied in the literature, but the occurrence condition of the multiple critical depths has not yet been determined. It should be stated that the main focus of previous studies has been the compound channels with horizontal floodplains; however, horizontal floodplains rarely occur in river channels. This is only an approach to simulate nature. In nature, floodplains are inclined. It is clear that the inclined floodplains represent real conditions much better than the horizontal floodplains.

In the present study, straight symmetrical prismatic channels with two different side slopes (compound triangular, rectangular, and parabolic sections; see Figs. 1–3) are considered, and the condition for which multiple critical depths exist is mathematically determined. For this purpose, limiting side slope for the upper portion of the cross section is analytically determined. If side slope of the upper portion is less than this limit, the multiple critical depths will occur in the two-stage cross section (also commonly known as compound channels).

This research explores the occurrence condition of the multiple critical depths in two-stage channels (main channel with flanking floodplain) and presents clear understanding and interpretation of this phenomenon. The following sections present the governing critical equation, the geometric properties of the considered two-stage cross sections, and the proposed procedure for determining the occurrence condition of multiple critical depths for compound channels with inclined floodplains.

Governing Equation for Critical Flow Depth

Critical depth is traditionally predicted on the basis of minimum specific energy for a simple channel. The flow depth corresponding to the minimum specific energy for a given discharge in an open channel is known as the critical flow depth. Some researchers have confirmed that this definition is not sufficient to determine

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