Alternative Solutions for Horizontal Circular Curves by Noniterative Methods

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Abstract: The most common type of horizontal curves used to connect intersecting straight sections of highways and other infrastructures are circular curves. Given the radius and the deflection angle of a horizontal circular curve, the other curve elements can be explicitly determined. However, there are 10 practical situations in which these parameters are unknown, and they have to be determined from two other given elements. In the present paper, these cases are classified into three groups according to their solution type. In the first group, direct analytical solutions can be easily obtained using simple algebraic operations. In the second group, full analytical solutions with physical meaning are not available, and in the third group, existing methods rely on a tedious trial procedure for most cases. The paper develops noniterative exact solutions for the two cases of the second group and noniterative near-exact solutions for the four cases of the third group. The proposed near-exact solutions for the third group, which have a maximum percentage error less than $2.5 \times 10^{-4}\%$, facilitate the design and analysis of horizontal circular curves. **DOI: 10.1061/(ASCE)SU.1943-5428.0000100.** © *2013 American Society of Civil Engineers.*

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Introduction

Horizontal circular curves have many practical applications such as surveying and mapping (Anderson and Mikhail 1998; Mannering and Kilareski 2005) and computer-aided geometric design (Farin 1988; Easa 2002). They are also used in railways, roadways, canals, and pipelines. In the geometric design of new horizontal curves, the curve radius and deflection angle are given, and all other curve elements can then be calculated using simple formulas. However, for existing curves that require reconstruction, the curve radius and deflection angle are not normally known and must be calculated using other curve elements.

For some cases, the governing equations of the horizontal circular curve are implicit, and hence they have to be solved using suitable methods. Because of their implicit forms, the solutions of these equations have received considerable attention by various researchers. Chen and Hwang (1992) presented iterative solutions of six circular curve cases using the Newton-Raphson (NR) method. Li (1993) proposed the iteration method, which requires no derivatives, where the iteration function is expressed in terms of the deflection angle. He also presented initial approximations of the deflection angle and a compound iterative scheme that enable faster conversion. Easa (1993) showed that the NR method used by Chen and Hwang (1992) needs an initial guess value of the curve radius and may not converge for some initial guess values. Subsequently, Easa (1994) proposed the iteration method where the iteration function is expressed in terms of the curve radius. Li (1995) explained the compound iterative scheme that makes the iterative process more efficient than the simple iteration method. Dubeau (1995a) showed that in general the NR method converges faster than the iteration method and suggested Aitken's acceleration technique to speed up the convergence of the iteration method. Easa (1995) presented the iteration functions for the multiple-solution case of the iteration method and showed the possible existence of multiple solutions for one case.

Dubeau (1995b) provided complete solutions of the problem of solving horizontal curves for small deflection angles ($0 < \Delta < \pi$), based on the iteration method by Li (1993), along with the conditions that the data must satisfy to ensure the existence of a solution and the initial approximation and the speed of convergence. He also presented direct solutions for the cases in which the deflection angle or the curve radius is known. This analysis for small deflection angles was later extended by Dubeau (1996) for large deflection angles ($\pi < \Delta < 2\pi$). Dubeau (1997) showed that a cubic function can be used to solve the cases when the middle ordinate and the tangent distance or the external distance and the long chord are known. The conditions that the data must satisfy to ensure the existence of a solution for cases are presented.

Khalil (2003) used a general cubic equation to solve some cases, but he did not develop physically meaningful (acceptable) solutions to cover the entire practical range. A direct solution of the case where the tangent length and curve length are known has been presented by Shebl and Alsaleh (2009). The authors used a cubic polynomial equation that characterizes the curve deflection angle and used the least-squares technique to facilitate the solution.

The preceding review shows that previous research on horizontal circular curves has mostly focused on numerical iterative methods for solving horizontal circular curves. Despite the many investigations in this field, there are no exact analytical solutions for all the cases in which the governing equations are not one-to-one. Moreover, when the governing equations are one-to-one, there are no

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