



## CIVIL ENGINEERING

# Accurate gradually varied flow model for water surface profile in circular channels

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**Abstract** The paper presents an accurate approximation of the Froude number ( $F$ ) for circular channels which is part of the gradually varied flow (GVF) equation. The proposed approximation is developed using optimization technique to minimize the relative error between the exact and estimated values, resulting in a maximum error of 0.6% compared with 14% for the existing approximate method. The approximate  $F$  is used in the governing GVF equation to develop an exact analytical solution of this equation using the concept of simplest partial fractions. A comparison of the proposed and approximate solutions for backwater length shows that the error of the existing approximate solution could reach up to 30% for large normal flow depths.

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## 1. Introduction

Accurate estimation of the water surface profile in gradually varied flows is important in the planning and design of water works. Numerous studies on gradually varied flow (GVF) have been conducted for different types of open channels [1–11]. However, analytical and semi-analytical studies of GVF related to circular open channels are limited. Chow [12] developed a semi-analytical solution for circular channels using

constant hydraulic components. To improve Chow solution, Zaghoul [13] derived mathematical expressions for the hydraulic exponents as functions of the GVF depth using numerical integration. Keifer and Chu [14] and Nalluri and Tomlinson [15] used a method that did not include hydraulic components. Nalluri and Tomlinson [15] modified Keifer and Chu's method using the subtended angle at the center instead of the relative depth. Both solutions used tabulated flow functions and their interpolated values which are not vary accurate. Zaghoul [16] developed a computer model to compute the GVF length for circular sections. The results were presented in a tabular form. Zaghoul and Shahin [17] also developed a package using the Lotus 1-2-3 to calculate the geometric and hydraulic properties of an open circular channel. The results were expressed as dimensionless quantities and were presented in tabular and graphical forms. Zaghoul [18], using variable pipe roughness, developed a computer model to calculate the

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