Short communication

Improved explicit approximation of linear dispersion relationship for gravity waves: Comment on another discussion

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1. Introduction

The discussers thank the authors for using Newton’s method to obtain an accurate and explicit two-step solution to linear dispersion relationship. Some points, however, should be taken into account regarding the application of the Newton’s method.

An explicit two-step expression to approximately solve the wave number as a function of the wave period is proposed by the authors using Newton’s method. As mentioned by the authors, Newton’s method works very well for dispersion equation, if a good seed is given. Considering Beji’s (2013) approximation as a seed, the authors have obtained an accurate explicit two-step solution with percentage error less than 0.0000082%.

According to the authors’ opinion, this two-step approach is substantially more accurate than the one proposed by Vatankhah and Aghashariatmadari (2013), with an error of 0.0010%. Since proposed solution by the authors is an explicit two-step one, an appropriate comparison must be made using the same method.

Using the Newton’s method, one gets

\[ \mu = \frac{\mu_0^2 + \mu_0 \cosh^2(\mu_0)}{\mu_0 + 0.5 \sinh(2\mu_0)} \]  

(1)

where \( \mu \) is an initial guess for \( \mu \). The authors proposed Beji’s (2013) approximation as a seed

\[ \mu = \frac{\mu_0^2 + \mu_0^{g_0}}{\sqrt{\tanh \mu_0}}. \]  

(2)

Eq. (2) results in a two-step solution, which has maximum relative error less than 0.0000082%.

Vatankhah and Aghashariatmadari (2013) showed that one-step solution of Eq. (2) can be simplified and improved to a better accuracy. To further develop an improved two-step solution in this study, Eq. (2) is first modified using the curve fitting method as follows

\[ \mu_0 = \frac{\mu_0^2 + \mu_0^{g_0}}{\sqrt{\tanh \mu_0}}. \]  

(3)

where \( a, b, c \) and \( d \) are coefficients. To determine these coefficients, the percentage error (PE) of the dispersion parameter, \( \mu \) is considered as follows

\[ \text{PE} = \left( \frac{\mu_0^{2+g_0} \cosh^2(\mu_0)}{\mu_0 + 0.5 \sinh(2\mu_0)} - 1 \right) \times 100 \]  

(4)

in which \( \mu_0 = \mu \tanh \mu \) and \( \mu \) is determined by Eq. (3). Then, the summations of PE values, that is, Sum iPEi is minimized as an objective