Technical Note

Evaluation of Explicit Numerical Solution Methods of the Muskingum Model

Ali R. Vatankhah[1]

Abstract: The nonlinear Muskingum model is a frequently used hydrologic routing method. In this model, the rate of change of the storage volume with respect to time is expressed by an ordinary first-order differential equation. Generally, this equation has no analytical solution, and thus, should be solved by standard numerical solution methods. Although many optimization techniques have been employed to estimate the parameters for the model, an accurate solution method for calculating the storage time variation of the Muskingum model is still required. Most previous researchers have used an inaccurate explicit Euler's method along with a manipulated routing equation for calculating the discharge at the downstream end to achieve a better fit for observed data. This manipulation, however, is not acceptable from a mathematical viewpoint. Until now, the storage time variation of the Muskingum model has only been calculated by an explicit Euler's method; other explicit numerical solution methods have not been used or clearly discussed. All explicit solution methods may produce similar results for small sized time intervals, but in practice for historical field data, the size of the time interval is fixed and insufficiently small; thus, a suitable solution method with sufficient accuracy should be used. This study proposes a fourth-order Runge-Kutta method as an accurate and suitable explicit solution method for calculating the storage time variation of the Muskingum model. **DOI: 10.1061/(ASCE)HE.1943-5584.0000978.**

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Introduction

Flood routing is useful for the study of flood wave propagation, particularly if the shape and peak of the hydrograph are required. The Muskingum model is a popular method for flood routing, which was first developed by the U.S. Army Corps of Engineers (McCarthy 1938). The routing procedure using the Muskingum model is based on continuity and storage equations. In the Muskingum model, it is assumed that the overall flood attenuating properties of the river reach remain constant and independent of the flooding event (Das 2009). Thus, the flood attenuating properties determined by using a historic set of observed inflow–outflow hydrographs (captured by the model parameters) can be used for routing any future outflow hydrographs.

The procedure for applying the Muskingum model involves two basic steps: a calibration or optimization step and a prediction or simulation step. The calibration or optimization step uses a method for calibrating hydrologic parameters (Luo and Xie 2010; Barati 2011; Xu et al. 2012). In the calibration step, a parameter estimation problem is solved in which the model parameters are determined by using inflow–outflow hydrograph data of the river reach under consideration. In the simulation step, a routing problem is numerically solved in which the outflow hydrograph for a given inflow hydrograph is determined by using a storage–discharge relationship, along with a one-dimensional continuity equation as a governing equation (Chow 1959; Gill 1978; Das 2004). In both steps (calibration and prediction), an accurate numerical method for solving the governing equations is required, which significantly affects the simulation results of the Muskingum model.

The Muskingum model has received considerable attention and has been the subject of many investigations. The linear Muskingum model may be unsuitable for the nonlinear relationship between the storage and discharge in most actual river systems. Earlier researchers have primarily focused on robust optimization algorithms to improve the parameter estimation of the nonlinear Muskingum model. Generally, the improvements attained by these techniques are insubstantial. An accurate solution method of the nonlinear Muskingum model is essential to the parameter estimation of the model (Vatankhah 2010). Most researchers have used an inaccurate Euler's solution method along with the manipulated equation for calculating the discharge to achieve a better fit for data observed by Wilson (1974). This manipulation is not acceptable from a mathematical viewpoint.

The nonlinear Muskingum model ends up an ordinary first-order differential equation that presents the rate of change in the storage with respect to time. An analytical solution of this differential equation is impossible; thus, this equation should be solved by using standard numerical solution methods. Most previous studies are limited to Tung's method, which is based on an inaccurate explicit Euler's solution (Tung 1985; Mohan 1997; Kim et al. 2001; Geem 2006; Luo and Xie 2010; Barati 2011; Geem 2014; Xu et al. 2012; Karahan et al. 2013; Orouji et al. 2013; Easa 2013a, b).

This research does not consider implicit numerical solution methods such as the Lagrange multiplier method (Das 2004). Instead, this research has focused on various explicit numerical solution methods for solving the Muskingum model and proposes the fourth-order Runge-Kutta method as an accurate and suitable solution method among the explicit solution methods for calculating the storage time variation of the Muskingum model. For this, a new criterion is also introduced to evaluate the accuracy of the explicit numerical solution methods.

[1]Assistant Professor, Dept. of Irrigation and Reclamation Engineering, Univ. College of Agriculture and Natural Resources, Univ. of Tehran, P.O. Box 4111, Karaj 31587-77871, Iran. E-mail: arvatan@ut.ac.ir

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